conditions of equilibrium of mixtures of alcohols and mineral acids. He considers in detail the velocity, and limits of etherification of the more important mineral acids, and arrives at many valuable results.

GEOGRAPHICAL NOTES

AT the last meeting of the Berlin Geographical Society news was received through a German trade house in Tangier that Dr. Lenz had reached Timbuctoo, and that he hoped to be at St. Louis, in Senegal, in the month of July. If this is correct Dr. Lenz has made a rapid journey in this direction, as he only left Tangier on December 22 last. Caillé, however, in 1828, travelled from Timbuctoo to Fez in four months. The last letter received from him by the Society was from Tenduf, in the beginning of May, twenty days' journey from Timbuctoo.

THE *Zeitschrift* of the Berlin Geographical Society, Nos. 88, 89, has a valuable map by Herr Richard Kiepert, showing the work done in Angola in 1876 by Dr. H. von Barth in the region of the Bengo and Lucalla, and of Herr Otto Schütt in 1877–79 on the Lower Quanza. Dr. von Möllendorff discusses the methods of transcribing Chinese geographical names, and concludes that the Pekin form of the Guan-hua, or so-called Mandarin dialect, would be best for general purposes. But Dr. von Möllendorff asks whether, while selecting this form generally, it is advisable to make exceptions in certain cases. Such names, for example, as already exist in familiar forms might be excepted, as Pekin, Canton, Hongkong, Swatow, &c. With other names, especially for special maps, a change from the uniform method of writing might be adopted. Maps of districts for the use of travellers would evidently be of greatest service when the local forms of names were given. Perhaps the Guan-hua might be used for the names of great towns, large rivers, and mountains, while smaller places might have the local forms of their names. For a map of the whole of China, or of the greater part of it, containing little more than the district towns, evidently the Guan-hua would be the preferable form. In books it would perhaps be best to give both forms. It is, no doubt, high time that some attempt at uniformity should be made, but the difficulty is by no means easy of solution, owing partly to the letters of the alphabet not being sounded uniformly in all European languages. Herr von Möllendorff instances the absurdity of the present want of system by the ways in which the Chinese name of the Yellow River is spelled. These are confusing enough, but what will he say when he sees "Houan Hé" (for Hwang-ho) at the head of the interesting communication just received from Col. Prejevalsky? We cannot entirely concur in Herr von Möllendorff's definition of "Kwan-hwa," popularly translated "Mandarin dialect," and he himself makes the orthographical jumble much worse by writing "Guan-hua," which we should imagine few sinologues would attempt to defend. The vexed question, however, may find a solution before long in an unexpected quarter, for the Statistical Department of the Chinese Maritime Customs at Shanghai, we believe, have under consideration a system of spelling for adoption in their reports and other publications, and this, if adopted, will probably come by degrees into general use. Dr. Hildebrandt gives an account of a visit he made to the Amber Mountains in the north of Madagascar; Herr K. Himy continues his elaborate paper on the region around the Kara-Kul, and much of the number is occupied with the journal in North Africa of the late Dr. Erwin von Bary.

THE new number of the Lyons Geographical Society's *Bulletin* contains several items of interest. M. Morice's paper on French Cochin-China is published with a sketch map, followed by some notes by the Abbé Desgodins on the hydrography and orography of Tibet, and a communication by the Abbé Faure on Potosi in Bolivia. Among the other contents are Père Brucker's notes on the geographical positions in Eastern Turkistan and Jungaria determined in 1876 by two Jesuit missionaries, and the report on Col. Flatters' explorations in the Central Sahara last spring.

M. VENUIKOFF has just published at Geneva an historical sketch of the geographical discoveries made in Asiatic Russia from the most remote times to our own days, illustrated by Perthes' map of North and Central Asia.

THE China Inland Mission have been informed by Mr. Samuel Clarke, one of their agents in the Chinese province of Szechuen, that, in company with Mr. Mollman, of the British and Foreign Bible Society, he lately made a journey from Chungking, on the Upper Yangtsze-kiang, to Chĕngtu-fu, the capital of the province, on which he travelled by unfrequented roads, where, so far as he could learn, no foreigner had ever been seen before; several previously unvisited towns were also entered. Mr. Clarke calls especial attention to the commercial activity prevailing along his route, and the frequency with which markets were held.

THE Asiatic Society of Bengal have just published, as an extra part of their *Journal*, a "Vocabulary of the Language of Eastern Turkistan," by the late Mr. R. B. Shaw, the well-known traveller, supplemented by two Turki vocabularies of birds and plants by Mr. J. Scully, lately on special duty at Kashgar.

FROM the Vienna *Allgemeine Zeitung* we gather that Dr. Emil Holub contemplates undertaking another lengthened journey in Central South Africa, provided that he can obtain the necessary funds. It is estimated that 50,000 florins will be required for the purpose, and it is proposed to raise this sum by a public subscription, the Austrian Geographical Society heading the list.

THE September number of the *Boletin* of the Madrid Geographical Society contains a detailed account of the Marquesas Islands, with map, by D. Ricardo Beltrán of Rózpide.

IT is stated that the *Gulnare*, with Capt. Howgate's expedition, landed at Rittenbank in Greenland, Dr. Pavy and Mr. Clay, whose intention is stated to be to make natural history collections and explore the northern limits of Greenland. This, we believe, is the same M. Pavy (a Frenchman) whose projected polar expedition suddenly collapsed in San Francisco seven years ago.

THE Austrian *Monatsschrift für den Orient* for October contains an article by Prof. Vambéry on the commercial importance of the Upper Oxus, in which he endeavours to show that there, and not on the Lower Oxus, is trade likely to be developed. Dr. Paulitschke gives an interesting sketch of the progress of African exploration during the past seventy years.

ON MAXIMUM AND MINIMUM ENERGY IN VORTEX MOTION[1]

I. A FINITE volume of incompressible inviscid fluid being given, in motion, filling a fixed, simply continuous, rigid boundary, the fact of its being in motion implies molecular rotation, or (as it may be called for brevity) vorticity. Helmholtz's law of conservation of vorticity shows that, whether the boundary be kept fixed as given, or be moved or deformed in any way, and brought back to its given shape and position, there remains in every portion of the fluid which had molecular rotation a definite constant of vorticity; and his formula for calculating energy for any given distribution of vorticity allows us to see that the energy may be varied by the supposed operation on the boundary.

II. The condition for steady motion of an incompressible inviscid fluid filling a finite fixed portion of space (that is to say motion in which the velocity and direction of motion continue unchanged at every point of the space within which the fluid is placed) is that, with given vorticity, the energy is a thorough maximum, or a thorough minimum, or a minimax. The farther condititon of *stability* is secured by the consideration of energy alone for any case of steady motion, for which the energy is a thorough maximum or a thorough minimum; because when the boundary is held fixed the energy is of necessity constant. But the mere consideration of energy does not decide the question of stability for any case of steady motion in which the energy is a minimax.

III. It is clear that, commencing with *any* given motion, the energy may be increased indefinitely by properly-designed operation on the boundary (understood that the primitive boundary is returned to). Hence, with given vorticity, there is no thorough maximum of energy in any case. There may also be *complete annulment* of the energy by operation on the boundary (with return to the primitive boundary), as we see by the following illustrations:—

1. The case of two equal, parallel, and oppositely rotating vortex columns terminated perpendicularly by two fixed parallel planes, which, by proper operation on the boundary, may be so

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[1] By Sir William Thomson, British Association, Swansea, Section A, Saturday, August 28.

mixed (like two eggs "whipped" together) that, infinitely near to any portion of either, there shall be some of the other.

2. The case of a single Helmholtz ring, reduced by diminution of its aperture to an infinitely long tube coiled within the inclosure.

3. The case of a single vortex column, with two ends on the boundary, bent till its middle meets the boundary; and farther bent and extended, till it is broken into two equal and opposite vortex columns; and then farther dealt with till these two are whipped together to mutual annihilation.

IV. To avoid for the present the extremely difficult general question illustrated (or suggested) by the consideration of such cases, confine ourselves now to two-dimensional motions in a space bounded by two fixed parallel planes and a closed cylindric surface perpendicular to them, subjected to changes of figure (but always truly cylindric and perpendicular to the planes). It is obvious that, with the limitation to two-dimensional motion, the energy cannot be either infinitely small or infinitely great with any given vorticity and given cylindric figure. Hence, under the given conditions, there certainly are at least two stable steady motions. We shall, however, see further that possibly in every case except cases of a narrow, well-defined character, and certainly in many cases, there is an infinite number of stable steady motions.

V. In the present case, clearly, though there are an infinite number of unstable steady motions, there are only two stable steady motions—those of absolute maximum and of absolute minimum energy.

VI. In every steady motion, when the boundary is circular, the stream lines are concentric circles, and the fluid is distributed in co-axial cylindric layers of equal vorticity. In the stable motion of maximum energy, the vorticity is greatest at the axis of the cylinder, and is less and less outwards to the circumference. In the stable motion of minimum energy the vorticity is smallest at the axis, and greater and greater outwards to the circumference. To express the conditions symbolically, let T be the velocity of the fluid at distance r from the axis (understood that the direction of the motion is perpendicular to the direction of r); the vorticity at distance r is—

$$\tfrac{1}{2}\left(\frac{T}{r}+\frac{dT}{dr}\right).$$

If the value of this expression diminishes from $r = 0$ to $r = a$, the motion is stable, and of maximum energy. If it increases from $r = 0$ to $r = a$ the motion is stable and of minimum energy. If it increases and diminishes, or diminishes and increases, as r increases continuously, the motion is unstable.

VII. As a simplest subcase, let the vorticity be uniform through a given portion of the whole fluid, and zero through the remainder. In the stable motion of greatest energy, the portion of fluid having vorticity will be in the shape of a circular cylinder rotating like a solid round its own axis, coinciding with the axis of the inclosure; and the remainder of the fluid will revolve irrotationally around it, so as to fulfil the condition of no finite slip at the cylindric interface between the rotational and irrotational portions of the fluid. The expression for this motion in symbols is:—

$$T = \zeta r \text{ from } r = 0 \text{ to } r = b;$$
$$\text{and } T = \frac{\zeta b^2}{r} \text{ from } r = b \text{ to } r = a.$$

VIII. In the stable motion of minimum energy the rotational portion of the fluid is in the shape of a cylindric shell, inclosing the irrotational remainder, which in this case is at rest. The symbolical expression for this motion is—

$$T = 0, \text{ when } r < \sqrt{(a^2 - b^2)} \text{ and } T = \zeta\left(r - \frac{a^2 - b^2}{r}\right),$$
$$\text{when } r > \sqrt{(a^2 - b^2)}.$$

IX. Let now the liquid be given in the configuration VII. of greatest energy, and let the cylindrical boundary be a sheet of a real elastic solid, such as sheet-metal with the kind of dereliction from perfectness of elasticity which real elastic solids present; that is to say, let its shape when at rest be a function of the stress applied to it, but let there be a resistance to change of shape depending on the velocity of the change. Let the unstressed shape be truly circular, and let it be capable of slight deformations from the circular figure in cross section, but let it always remain truly cylindrical. Let now the cylindric boundary be slightly deformed and left to itself, and held so as to prevent it from being carried round by the fluid. The central vortex column is set into vibration in such a manner that longer and shorter waves travel round it with less and greater angular velocity.[1] These waves cause corresponding waves of corrugation to travel round the cylindric bounding sheet, by which energy is consumed, and moment of momentum taken out of the fluid. Let this process go on until a certain quantity of moment of momentum has been stopped from the fluid, and now let the canister run round freely in space, and, for simplicity, suppose its material to be devoid of inertia. The whole moment of momentum is initially—

$$\pi\zeta b^2(a^2 - \tfrac{1}{2}b^2);$$

It is now

$$\pi\zeta b^2(a^2 - \tfrac{1}{2}b^2) - M,$$

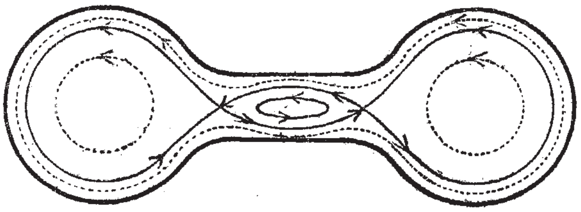
and continues constantly of this amount as long as the boundary is left free in space. The consumption of energy still goes on, and the way in which it goes on is this: the waves of shorter length are indefinitely multiplied and exalted till their crests run out into fine laminæ of liquid, and those of greater length are abated. Thus a certain portion of the irrotationally revolving water becomes mingled with the central vortex column. The process goes on until what may be called a vortex sponge is formed; a mixture homogeneous on a large scale, but consisting of portions of rotational and irrotational fluid, more and more finely mixed together as time advances. The mixture is, as indicated above, altogether analogous to the mixture of the substances of two eggs whipped together in the well-known culinary operation. Let b' be the radius of the cylindric vortex sponge, b being as before the radius of the original vortex column—

$$\tfrac{1}{2}b'^2 = \tfrac{1}{2}b^2 + \frac{M}{\zeta b^2}.$$

X. Once more, hold the cylindric case from going round in space, and continue holding it until some more moment of momentum is stopped from the fluid. Then leave it to itself again. The vortex sponge will swell by the mingling with it of an additional portion of irrotational liquid. Continue this process until the sponge occupies the whole inclosure.

After that continue the process further, and the result will be that each time the containing canister is allowed to go round freely in space, the fluid will tend to a condition in which a certain portion of the original vortex core gets filtered into a position next to the boundary, and the fluid within it tends to a more and more nearly uniform mixture of vortex with irrotational fluid. This central vortex-sponge, on repetition of the process of preventing the canister from going round, and again leaving it free to go round, becomes more and more nearly irrotational fluid, and the outer belt of pure vortex becomes thicker and thicker. The final condition towards which the whole tends is a belt constituted of the original vortex core now next the boundary; and the fluid which originally revolved irrotationally round it now placed at rest within it, being the condition (VIII. above) of absolute minimum energy. Begin once more with the condition (VII. above) of absolute maximum energy, and leave the fluid to itself, whether with the canister free to go round sometimes, or always held fixed, provided only it is ultimately held from going round in space; the ultimate condition is always the same, viz., the condition (VIII.) of absolute minimum energy.

XI. That there may be an infinite number of configurations of stable motions, each of them having the energy of a thorough minimum as said in IV. above, we see, by considering the case, in which the cylindric boundary of the containing canister consists of two wide portions communicating by a narrow passage, as shown in the sketch. If such a canister be completely filled with irrotationally moving fluid of uniform vorticity, the stream-lines must be something like those indicated in the sketch.



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[1] See *Proceedings* of the Royal Society of Edinburgh for 1880, or *Philosophical Magazine* for 1880; Vibrations of a Columnar Vortex; Wm Thomson.

Hence if a small proportion of the whole fluid is irrotational, it is clear that there may be a minimum energy, and therefore a stable configuration of motion with the whole of this in one of the wide parts of the canister; or the whole in the other; or any proportion in one and the rest in the other; or a small portion in the elliptic whirl in the connecting canal, and the rest divided in any proportion between the two wide parts of the canister.

ON THE SPECTRA OF THE COMPOUNDS OF CARBON WITH HYDROGEN AND NITROGEN

MESSRS. LIVEING AND DEWAR have made a long series of observations on this subject, of which the following is a brief abstract[1] by the authors:—

The first experiments were made with a De Meritens dynamo-electric machine, arranged for high tension, giving an alternating current capable of producing an arc between carbon poles in air of from 8 to 10 millims. in length. The carbon poles used had been previously purified by prolonged heating in a current of chlorine.

The arc was taken in different gases inside a small glass globe about 60 millims. in diameter, blown in the middle of a tube. The two ends of the tube were closed with dry corks, through which were passed (1) the carbons inserted through two pieces of narrow glass tubing; (2) two other glass tubes through which currents of the gases experimented with were sent.

The arc taken in a globe of air gave a tolerably bright continuous spectrum, above which the green and blue hydrocarbon bands were seen, also the seven bands in the indigo (wave-lengths 4,600 to 4,502, Watts) as in the flame of cyanogen, and much more brightly the six bands in the violet (wave-lengths 4,220 to 4,158, Watts) and five ultra-violet.

Carbonic acid gas, hydrogen, nitrogen, chlorine, carbonic oxide, nitric oxide, and ammonia were then successively substituted for air in the globe, with the result that in carbonic acid, hydrogen, chlorine, and carbonic oxide, the above-mentioned bands in the indigo, violet, and ultra-violet died away, while in nitrogen, nitric oxide, and ammonia, they were always well seen.

These different gases were used in order to eliminate to a large extent the influence of electric conductivity on the character of the spectrum; and the green and blue hydrocarbon bands were seen, more or less, in all of them.

Next observations were made of the spectra of flames of sundry compounds of carbon.

In the flame of cyanogen, prepared from well-dried mercury cyanide, passed over phosphoric anhydride inserted in the same tube, and burnt from a platinum jet fused into the end of the tube, the hydrocarbon bands were almost entirely absent, as Plücker and Hittorf had found; only the brightest green band was seen, and that faintly. The indigo, violet, and ultra-violet bands, on the other hand, were well developed.

These three sets of bands in the indigo, violet, and ultra-violet are in the sequel referred to as the "cyanogen bands," though it is possible that they may be producible by other compounds of carbon with nitrogen.

The flame of hydrocyanic acid burning in air showed very much the same as that of cyanogen.

In the flame of a mixture of purified hydrogen and carbon disulphide no hydrocarbon bands at all could be detected.

Nor could they be detected in the flame of a mixture of carbonic oxide and hydrogen burnt in air.

When a mixture of hydrogen or of carbonic oxide with carbon tetrachloride vapour was burnt, hydrocarbon bands made their appearance, but were weak.

On the other hand, chloroform, when mixed with hydrogen, gave, when burnt in air, the hydrocarbon bands very strongly.

On a review of the whole series of observations, certain points stand out plainly. In the first place, the indigo, violet, and ultra-violet bands, characteristic of the flame of cyanogen, are conspicuous in the arc taken in an atmosphere of nitrogen, air, nitric oxide, or ammonia, and they disappear almost, if not quite, when the arc is taken in a non-nitrogenous atmosphere of hydrogen, carbonic oxide, carbonic acid, or chlorine. These same bands are seen brightly in the flames of cyanogen and hydrocyanic acid, but are not seen in those of hydrocarbons, carbonic oxide, or carbon disulphide. The conclusion seems irresistible that they belong to cyanogen; and this conclusion does not seem to be at all invalidated by the fact that they are seen weakly, or by flashes, in the arc or spark taken in gases supposed free from nitrogen by reason of the extreme difficulty of removing the last traces of air. They are never, in such a case, the principal or prominent part of the spectrum, and in a continuous experiment they are seen to fade out in proportion as the nitrogen is removed. This conclusion is strengthened by the observations that cyanogen (or hydrocyanic acid) is generated in the arc in atmospheric air in large quantity.

In the next place, the green and blue bands, characteristic of the hydrocarbon flame, seem to be always present in the arc, whatever the atmosphere. This is what we should expect if they be due, as Ångström and Thalèn suppose, to acetylene; for the carbon electrodes always contain, even when they have been long heated in chlorine, a notable quantity of hydrogen. In the flames of carbon compounds they by no means always appear; indeed it is only in those of hydrocarbons or their derivatives that they are well seen. Carbonic oxide and carbon disulphide, even when mixed with hydrogen, do not show them; and if seen in the flames of cyanogen, hydrocyanic acid, and carbon tetrachloride mixed with hydrogen, they are faint, and do not form a principal or prominent part of the spectrum. This is all consistent with the supposition of Ångström and Thalèn. The fact that the bands are not produced even in the presence of hydrogen, when it is not present in the flame in the form of a compound with carbon, is very significant; for we know that acetylene is present, and can easily be extracted from the flame of any hydrocarbon, and that it is formed as a proximate product of decomposition of hydrocarbon by the electric discharge, but we have no evidence that it is producible as a product of direct combination of carbon with hydrogen at the comparatively low temperature of the flames described.

The hydrocarbon bands are best developed in the blowpipe flame, that is under conditions which appear, at first sight, unfavourable to the existence of acetylene in the flame. However, by the use of a Deville's tube, acetylene may readily be withdrawn from the interior of such a flame, and from that part of it which shows the hydrocarbon bands most brightly.

The question as to whether these bands are due to carbon itself or to a compound of carbon with hydrogen, has been somewhat simplified by the observations of Watts, Salet, and others on the spectrum of carbonic oxide. It can hardly be doubted now that that compound has its own spectrum quite distinct from the hydrocarbon flame spectrum. The mere presence of the latter spectrum feebly developed in the electric discharge in compounds of carbon supposed to contain no hydrogen, weighs very little against the series of observations which connect this spectrum directly with hydrocarbons.

In the next place, it appears conclusively from the experiments, that the development of violet bands of cyanogen, or the less refrangible hydrocarbon bands, is not a matter of temperature only. For the appearance of the hydrogen lines C and F, observed by the authors in the arc taken in hydrogen, indicates a temperature far higher than that of any flame. Yet the violet bands are not seen in hydrogen at that temperature, while the green bands are well developed. The violet bands are, nevertheless, seen equally well at the different temperatures of the flame, arc, and spark, provided cyanogen be the compound under observation in the flame, and nitrogen and carbon are present together at the higher temperatures of the arc and spark.

The accompanying diagram (Fig. 1) shows approximately the relative position of the bands in that part of the spectrum of the flame of cyanogen fed with a jet of oxygen which is more refrangible than the Fraunhofer line F. Only those bands which are less refrangible than the solar line L have been before described, but photographs show two shaded bands slightly less refrangible than the solar line N accompanied by a very broad diffuse band of less intensity on the more refrangible side of N; also a strong shaded band, which appears to be absolutely coincident with the remarkable shaded band in the solar spectrum, which has been designated by the letter P; and near this, on the less refrangible side, a much fainter diffuse band, which also seems to coincide with a part of the solar spectrum sensibly less luminous than the parts on either side of it. This spectrum is remarkably persistent at all temperatures of the flame. Watts found that it did not disappear when the flame was cooled down as much as possible by diluting the cyanogen with carbonic acid; it retains its characters when the cyanogen is burnt in nitric oxide. The flame in the last case must be one

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[1] For fuller details see *Proc.* R.S., xxx. pp. 152, 494.